CS3 Computability and Intractability (2001-2002)
Exercise Sheet 3

The deadline for this coursework is noon on Thursday 14 March. Please submit your solutions directly to your tutor. Solutions submitted after the deadline, but before noon on Monday 18 March, will have their credit reduced by one third. No credit will be given for solutions submitted after that date. Please note that multiple submissions are not allowed. The marks for questions are not always related to their length or difficulty. Answers that are not completely correct but show relevant reasoning will be awarded partial credit (depending on how much progress is shown).

1. (a) Show that if a language L reduces to $\emptyset$ then $L = \emptyset$. [*2 marks*]

   (b) Let L_{empty} be the language

   $$L_{\text{empty}} = \{\langle M \rangle \mid L(M) = \emptyset\}.$$

   Prove, with appropriate reference to the notes, that L_{empty} is not recursive. [*3 marks*]

   (c) Given two languages it would be useful if we could decide if one reduces to the other. Let

   $$L_{\text{red}} = \{\langle M_1 \rangle \$ \langle M_2 \rangle \mid \text{there is a reduction from } L(M_1) \text{ to } L(M_2)\}.$$

   Prove that L_{red} is not recursive. [*5 marks*]

2. The *Vertex Cover* problem, VC, is the following.

   Instance: A graph $G = (V, E)$ and integer k.
   Question: Does G contain a vertex cover of size k? (A *vertex cover* is a set $U \subseteq V$ such that every edge of G has at least one endpoint in U.)

   (a) Demonstrate that IndSet $\leq_{\text{P}}$ VC. [*5 marks*]

   (b) It is known that IndSet is NP-complete. Which additional fact would enable you to deduce that VC is also NP-complete? Provide a brief justification for this additional fact. [*2 marks*]

   (c) Is the following problem likely to be NP-complete? Justify your answer.

   Instance: A graph $G = (V, E)$.
   Question: Does G contain a vertex cover of size 10? [*3 marks*]

3. The problem k-2Sat is defined as follows.

   Instance: A Boolean formula ϕ in conjunctive normal form (CNF) such that each clause in ϕ has at most *two* literals, and an integer k.

QUESTION: Does there exist a satisfying assignment for ϕ that sets at least k of the variables occurring in ϕ to the value *true*?

By exhibiting a reduction from IndSet, or otherwise, prove that k-2Sat is NP-hard. [*10 marks*]

4. The following problem, TimeTable, arises in creating a timetable for examination papers.

   INSTANCE: A set P of papers, a set S of timetable "slots", a set $C = \{c_1, \ldots, c_k\}$ of candidates, and, for each candidate c_i, a set $P_i \subseteq P$ of papers that the candidate is expecting to sit.

   QUESTION: Is there an assignment of the examination papers to timetable slots that avoids clashes? (A clash occurs if some candidate is required to sit two papers simultaneously.)

   By presenting a reduction from Colourability (or otherwise) show that TimeTable is NP-hard. [*10 marks*]

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