UNIVERSITY OF EDINBURGH

FACULTY OF SCIENCE AND ENGINEERING

COMPUTER SCIENCE FOUR

MODELLING AND SIMULATION

Tuesday 17 April 2001

09:30 to 11:00

Examiners:

Prof. K. Turner (External)

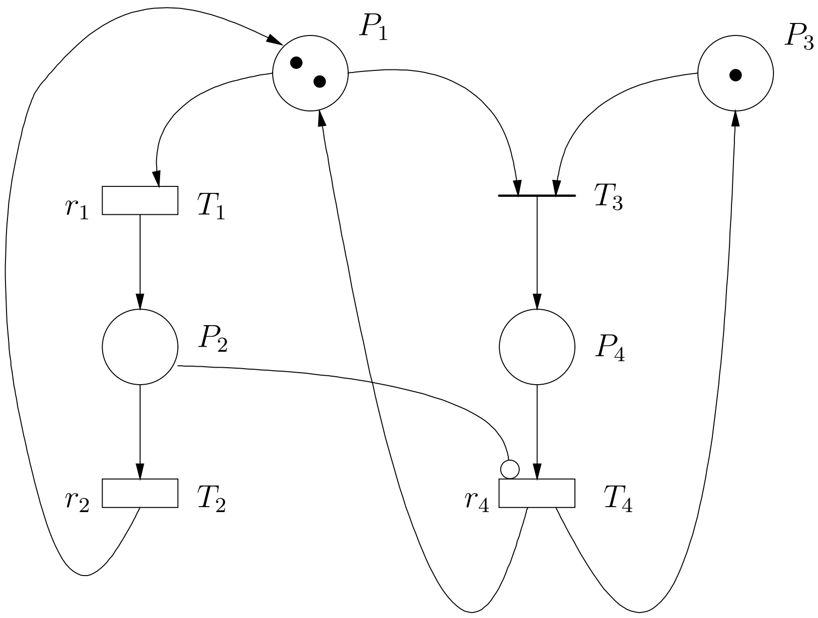
Prof. G. Plotkin (Chairman)

INSTRUCTIONS TO CANDIDATES

Answer any TWO questions.

All questions carry equal weight.

1. (a) Consider the GSPN shown below:



i. Draw the reachability graph of this net.

ii. Draw the reachability graph that would be obtained if the inhibitor arc were removed.

iii. Draw the reachability graph that would be obtained if, instead, T_3 became a timed transition.

iv. Describe in detail the transformations that must be made to the reachability graph of an arbitrary GSPN model in order to obtain the underlying Markov process.

v. Draw the state transition diagram of the Markov process corresponding to the GSPN model shown above (as it is depicted). [*12 marks*]

(b) In an arbitrary system let C denote the number of job completions observed during a time interval of length T. Similarly, let C_i denote the number of job completions observed at the i-th device or resource within the system during the same interval.

i. Use these variables to justify the *forced flow law*:

$$X_i = XV_i$$

ii. What assumptions do you need to make about the system? [*5 marks*]

QUESTION CONTINUES ON NEXT PAGE

(c) Consider an input/output subsystem that consists of two disks and the associated controller hardware and software. Disk A takes $500\mu s$ on average to process a request and Disk B takes $400\mu s$ on average. Disk A processes twice as many requests as Disk B. If Disk A is 50% utilised calculate the following measures:

i. the throughput of Disk A;
ii. the throughput of Disk B;
iii. the utilisation of Disk B.

[**Note:** You may assume that the assumptions necessary for application of operational analysis are valid for this system.] *[8 marks]*

2. (a) Explain what it means for a Markov process to have reached *steady state* or *equilibrium* behaviour. [*2 marks*]

(b) The matrix below is the infinitesimal generator matrix of a Markov process.

$$\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & -\frac{23}{24} & \frac{1}{3} & \frac{1}{8} & 0 \\ \frac{1}{4} & 0 & -\frac{3}{8} & 0 & \frac{1}{8} \\ 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & -\frac{1}{4} \end{pmatrix}$$

i. Draw the state transition diagram of a corresponding Markov process.

ii. Explain why your answer to part i. is not unique.

iii. What two transformations are needed if the process is to be solved for steady state using a linear algebra package such as maple? Explain why each transformation is necessary and illustrate it on the matrix above. [*10 marks*]

QUESTION CONTINUES ON NEXT PAGE

(c) In a transaction processing system on average each transaction requires four bursts of CPU and three database accesses, two to Disk 1 and one to Disk 2. The typical behaviour of a transaction is described by the following sequence.

- launched by user
- visit to CPU
- visit to Disk 1
- visit to CPU
- visit to Disk 2
- visit to CPU
- visit to Disk 1
- visit to CPU
- confirm to user.

After the launching of one transaction the user constructs another but does not launch it until the first has completed, even if ready to do so.

The system is to be modelled by a discrete event simulation model based on SimJava. The modeller has identified four entities to be used in the model: User, CPU, Disk 1 and Disk 2.

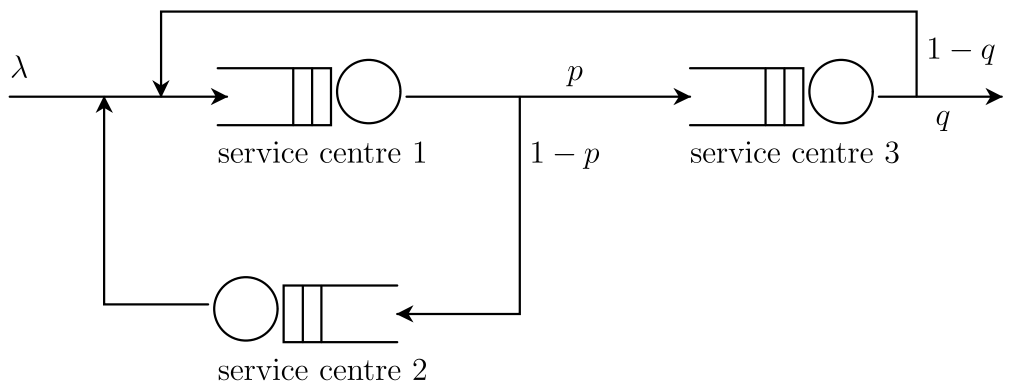
i. Explain what *events* are in SimJava models.
ii. Explain the SimJava events which would be needed to model the system above using the entities described, and detail the ports which would be needed in each entity to support them.
iii. Draw a diagram showing how the ports would be linked in the complete model if there are two users in the system.
iv. Describe two techniques the modeller could use to ensure that the data she collects represents steady state behaviour of the system. [*13 marks*]

3. (a) i. What is the *traffic intensity* of a single server queue?

ii. Consider two $M/M/1$ queues. The first has service rate μ and arrival rate $\lambda/2$; the second has service rate 2μ and arrival rate λ. How would the performance characteristics of the two queues compare? Justify your answer.

iii. Given the service rate of an $M/M/1$ queue, what can you conclude about the throughput of the queue? Justify your answer. [*6 marks*]

(b) Consider the queueing network shown below:



Given that $\lambda = 22$, $p = \frac{1}{3}$ and $q = \frac{1}{4}$, calculate the following rates:

i. the arrival rate at service centre 2;

ii. the departure rate from service centre 1;

iii. the departure rate from the system. [*6 marks*]

(c) Compositional approaches to modelling are based on the notion that elements of the system can be modelled separately and then combined to construct a model of the complete system. In general, in the complete system the separate elements will influence each other's behaviour by interacting.

i. What forms of interaction between entities can be represented in a PEPA model? Illustrate your answer by model fragments.

ii. How could these forms of interaction be represented in a SimJava model? [*13 marks*]