WHY SYSTOLIC ARRAYS

(the real answer)

or

Computing In The Polyhedral Model

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Tutorial Organization

- Introduction, Notation & Definitions
- PART I: Languages
  - The Alpha Language
  - The Alpha Transformation System (+ Demo)
- PART II: Scheduling
  - Foundations, Taxonomy, Validity Conditions
  - Optimal Schedules
- PART III:
  - Placement, Localization and Communication
  - Code Generation
- PART IV: Extensions, Limitations and Conclusion

Introduction

- Summary of Systolic Array Synthesis (history lesson)
- Recurrence Equations
- Domains, Polyhedra, Prameters

History Lesson

- Specify a recurrence equation
- Transform/massage it to have uniform dependencies
- Determine a schedule t: no computation can be scheduled before its arguments (respect causality)
- Choose an allocation function a: two points must not be mapped to the same processor at the same time (no conflict with schedule)

Recurrence Equations: Definition

$$X[z] = \{\ z \in D\} : \quad \Rightarrow \quad g(\ldots X[f(z)]\ldots)$$

- X n-dimensional data variable ;
- z n-dimensional index variable .
- g strict, ("atomic") function;
- $f(z)$ *dependency function*, $f : \mathcal{Z}^n \rightarrow \mathcal{Z}^n$;
- "$\ldots$" — other such arguments;
- $D \subseteq \mathcal{Z}^n$, domain of the equation.

Recurrence Equations: Taxonomy

- **Uniform Recurrence Equations** (URE) — $f(z)$ has the form $z + \delta$.

- **Affine Recurrence Equations** (ARE) — $f(z)$ is an affine function, $Az + a$.

- **System of Recurrence Equations** (SRE) — set of (mutually recursive) equations. Could be uniform or affine (SURE or SARE).

Domains, Polyhedra & Parameters

A domain is

- Set of index points in $\mathcal{Z}^n$ where the recurrence is defined (the mathematical *domain* of a function
- The iteration space of a loop program
- The extent of multidimensional arrays

Different families of domains:

- The positive orthant
- Any finite subset of $\mathcal{Z}^n$
- A polyhedral index space
- A parameterized family of polyhedra

Parameterized Polyhedral Domains

- Integral Polyhedron: (finite or infinte) set of points satisfying a finite number of linear inequalities

$$\{z \in \mathcal{Z}^n \mid Qz \geq q\}$$

- Parameterized family of polyhedra: the q may include a linear function of m size parameters, p

$$\{z \in \mathcal{Z}^n \mid Qz \geq q - Pp\}$$

Alternatively:

$$\left\{ \begin{pmatrix} z \\ p \end{pmatrix} \in \mathcal{Z}^{n+m} \mid \begin{bmatrix} Q & P \end{bmatrix} \begin{pmatrix} z \\ p \end{pmatrix} \geq q \right\}$$

Dual Representation of Polyhedra

- Generators: Set of convex combinations of a set of **vertices**

$$\{z \in \mathcal{Z}^n \mid z = a^T G; \ \sum_i a_i = 1\}$$

- Each Representations is useful in certain contexts.

Standard (self dual) algorithm to convert from one to other.