Functional In-place Update with Layered Datatype Sharing

Michal Konečný
LFCS, University of Edinburgh

part of a wider collaboration with
David Aspinall, Martin Hofmann, Robert Atkey

Overview

- *LFPL* (Linear Functional Programming Language) [Hofmann 2000]
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- resource type $\Diamond$ $\Rightarrow$ simple and efficient evaluation using heap
- *linear typing* guarantees correctness of this evaluation
  forbids sharing on the heap
- developing less restrictive typings for LFPL
  - *usage aspects* (Aspinall & Hofmann 2002)
  - *explicit sharing in the context* (Atkey)
  - *layered datatype sharing* (K 2002)
- Scope: first-order, full recursion, arbitrary inductive datatypes

Append in LFPL

In ML :

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\begin{aligned}
\mathit{append}(x, y) = \ & \text{match } x \text{ with} \\
& \quad \text{Nil} \to y \\
& \quad \mid \text{Cons}(h, t) \quad \to \text{Cons}(h, \mathit{append}(t, y))
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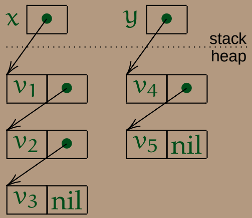
Heap representation:



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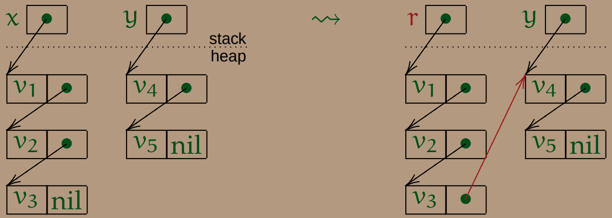

Non-linear typings

In LFPL:
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LFPL with Usage aspects (UAPL, Aspinall & Hofmann 2002):

$x :^1 \mathsf{L}(A), y :^2 \mathsf{L}(A) \vdash \mathit{append}(x, y) : \mathsf{L}(A)$ $\qquad x :^3 \mathsf{L}(A) \vdash \mathit{length}(x) : \mathsf{Nat}$

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expression	eval?	LFPL
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$\mathsf{Cons}(\mathit{length}(x), x)@d$	Y	N
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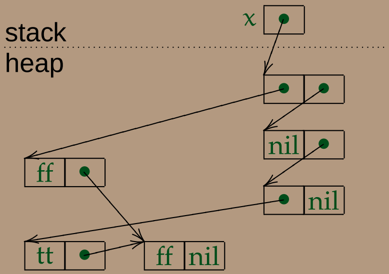
Datatype portions (layers)

$$\mathsf{L}(A) = \mu X.\mathsf{Unit} + {}^{\Diamond}(A \times X)$$



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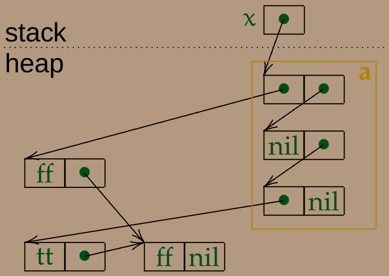

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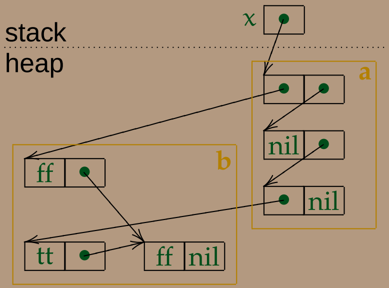

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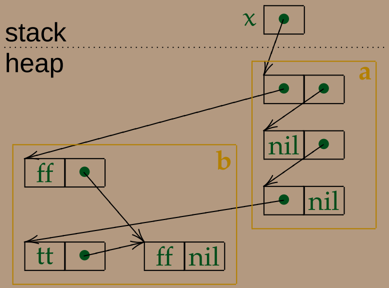

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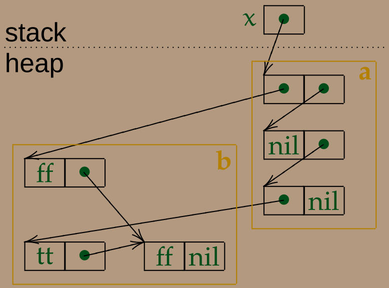

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Basic Separation Assertions

notation	type pattern	name
$\mathbf{a} \otimes \mathbf{b}$	$\ldots^{\Diamond[\mathbf{a}]} \ldots^{\Diamond[\mathbf{b}]} \ldots$	$\mathbf{a}$ separated from $\mathbf{b}$

Basic Separation Assertions

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$a \otimes b$	$\ldots \Diamond^{[a]} \ldots \Diamond^{[b]} \ldots$	a separated from b
$\otimes b / a$	$\ldots \mu X^{(a)}.(\ldots \Diamond^{[b]} \ldots) \ldots$	b separated along a
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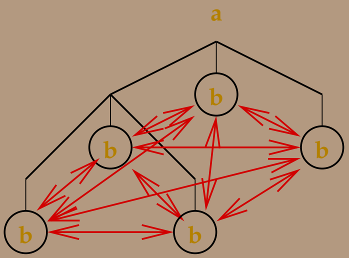

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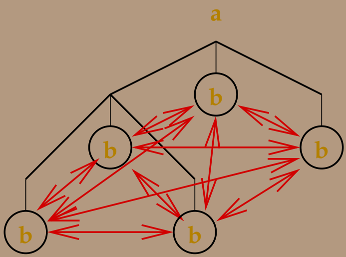

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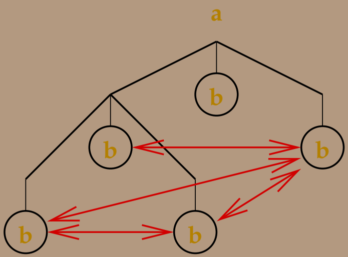
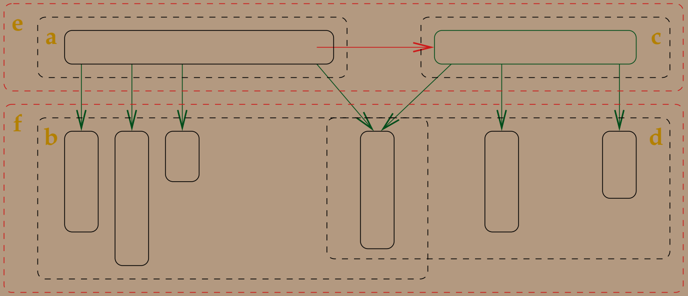


Typing of append



$x : \mathsf{L}^{[a]}(\mathsf{L}^{[b]}(\mathsf{Bool})), y : \mathsf{L}^{[c]}(\mathsf{L}^{[d]}(\mathsf{Bool}));$ argument portions

$\vdash$

$append(x, y) : \mathsf{L}^{[e\quad]}(\mathsf{L}^{[f\quad]}(\mathsf{Bool}));$ result portions

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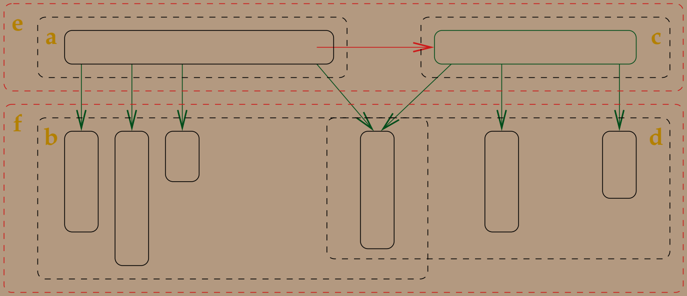

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$\{a\}$ destroyed portions

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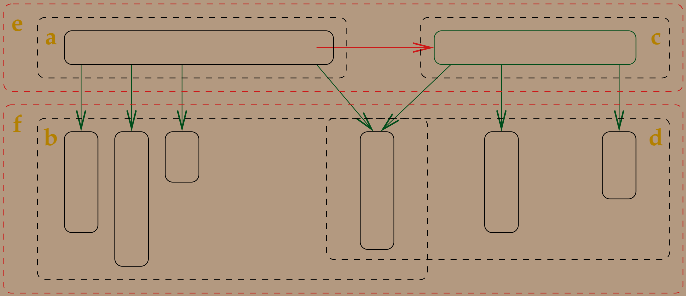

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$\mathit{append}(x, y) : \mathsf{L}^{[e \subseteq \{a,c\}]}(\mathsf{L}^{[f \subseteq \{b,d\}]}(\mathsf{Bool}));$ result portions, containment

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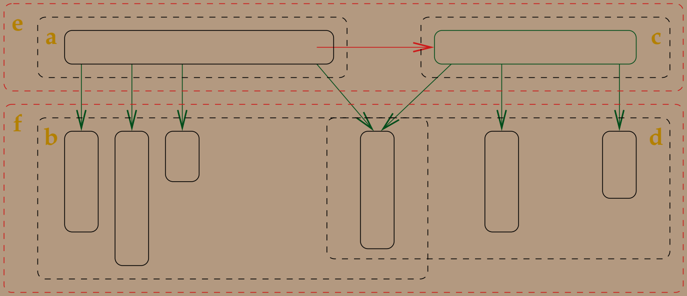
Typing of append



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$\{a \otimes b, a \otimes c, a \otimes d\} \vdash$ separation pre-condition

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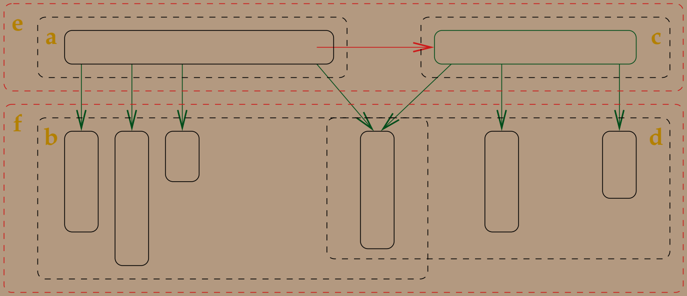

$x : L^{[a]}(L^{[b]}(\mathsf{Bool})), y : L^{[c]}(L^{[d]}(\mathsf{Bool}));$	argument portions
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$\{a\}$	destroyed portions
$\otimes f/e \Longleftarrow \{b \otimes d, \otimes b/a, \otimes d/c\}$;	separation rely-guarantees

A typing rule

[CONS]

$$B_h = A\left[\frac{\delta}{\rho}\right] \qquad B_t = \mathsf{L}^{[\zeta]}\left(A\left[\frac{\delta'}{\rho'}\right]\right)$$

$h : B_h, t : B_t, d : \Diamond^{[\zeta_d]};$

$\vdash$ *separation pre-condition*

$\mathsf{Cons}(h, t)@d$

$: \quad ;$ *containment guarantees*

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$\{\zeta_d\} \otimes \big(N_D(B_h) \cup N_D(B_t)\big) \vdash$ *separation pre-condition*

$\mathsf{Cons}(h, t)@d$

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A typing rule

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$$B_h = A\left[\tfrac{\delta}{\rho}\right] \qquad B_t = \mathsf{L}^{[\zeta]}\left(A\left[\tfrac{\delta'}{\rho'}\right]\right)$$

$$E = \mathsf{L}^{[\zeta' \subseteq \{\zeta_d\}]}(E_h) \cup E_t$$

$h : B_h, t : B_t, d : \Diamond^{[\zeta_d]};$

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$\{\zeta_d\} \ ;$ *destroyed portions*

G *separation rely-guarantees*

where $G = G_M(E) \cup G_Y(E_h, B_h) \cup G_Y(E_t, B_t)$
$\cup \left[\otimes \zeta_\xi / \zeta' \Longleftarrow \{\delta(\xi) \otimes \delta'(\xi)\}\right]_{\xi \in \mathrm{AddrD}(A)}$

Typing a recursive program

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- initial annotation - optimistic:

$x : \mathsf{L}^{[a]}(\mathsf{L}^{[b]}(\mathsf{Bool})), y : \mathsf{L}^{[c]}(\mathsf{L}^{[d]}(\mathsf{Bool}));$

$\varnothing \vdash$ *separation pre-condition*

match x with
  Nil $\rightarrow y$
  | Cons$(h, t)@d \rightarrow$ let $z =$ Cons$(h, y)@d$
    in $append(t, z)$

$: \mathsf{L}^{[e \subseteq \varnothing]}(\mathsf{L}^{[f \subseteq \varnothing]}(\mathsf{Bool}));$ *containment guarantees*

$\varnothing;$ *destroyed portions*

$\otimes \mathbf{f}/\mathbf{e} \Longleftarrow \varnothing$ *separation rely-guarantees*

Typing a recursive program

- annotating expressions - need annotated function symbols
- initial annotation - optimistic
- then annotate expression

$t : L^{[a]}(L^{[b]}(\mathsf{Bool})),\ z : L^{[c]}(L^{[d]}(\mathsf{Bool}));$

$\varnothing \vdash$ *separation pre-condition*

$\text{match } x \text{ with}$

$\quad \text{Nil} \to y$

$\quad | \text{ Cons}(h, t)@d \to \text{ let } z = \text{ Cons}(h, y)@d$

$\qquad\qquad\qquad\qquad \text{in } \mathit{append}(t, z)$

$: L^{[e \subseteq \varnothing]}(L^{[f \subseteq \varnothing]}(\mathsf{Bool}));$ *containment guarantees*

$\varnothing;$ *destroyed portions*

$\otimes \mathbf{f}/\mathbf{e} \Longleftarrow \varnothing$ *separation rely-guarantees*

Typing a recursive program

- annotating expressions - need annotated function symbols
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match x with

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$\{d\};$ *destroyed portions*

$\otimes f/e \Longleftarrow \{a \otimes c, \otimes c/b\}$ *separation rely-guarantees*

Typing a recursive program

- annotating expressions - need annotated function symbols
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$h : L^{[a]}(\mathrm{Bool}),\ y : L^{[b]}(L^{[c]}(\mathrm{Bool})),\ d : \Diamond^{[d]},\ t : L^{[e]}(L^{[f]}(\mathrm{Bool}))$

$\{a \otimes d, b \otimes d, c \otimes d, d \otimes e, d \otimes f\} \vdash$ *separation pre-condition*

match x with
Nil $\to y$
$|$ Cons$(h, t)@d \to$ let $z =$ Cons$(h, y)@d$
in $append(t, z)$

$: L^{[g \subseteq \emptyset]}(L^{[h \subseteq \emptyset]}(\mathrm{Bool}));$ *containment guarantees*

$\{d\};$ *destroyed portions*

$\otimes h/g \Longleftarrow \emptyset$ *separation rely-guarantees*

Typing a recursive program

- annotating expressions - need annotated function symbols
- initial annotation - optimistic
- then annotate expression

$x : \mathsf{L}^{[a]}(\mathsf{L}^{[b]}(\mathsf{Bool})), y : \mathsf{L}^{[c]}(\mathsf{L}^{[d]}(\mathsf{Bool}));$

$\{\mathbf{a} \otimes \mathbf{c}, \mathbf{a} \otimes \mathbf{d}\} \vdash$ *separation pre-condition*

match x with
  Nil $\rightarrow y$
  | Cons$(h, t)@d \rightarrow$ let $z =$ Cons$(h, y)@d$
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$: \mathsf{L}^{[e \subseteq \{c\}]}(\mathsf{L}^{[f \subseteq \{d\}]}(\mathsf{Bool}));$ *containment guarantees*

$\{\mathbf{a}\};$ *destroyed portions*

$\otimes \mathbf{f}/\mathbf{e} \Longleftarrow \{\otimes \mathbf{d}/\mathbf{c}\}$ *separation rely-guarantees*

Typing a recursive program

- annotating expressions - need annotated function symbols
- initial annotation - optimistic
- then annotate expression and repeat

$x : \mathsf{L}^{[a]}(\mathsf{L}^{[b]}(\mathsf{Bool})), y : \mathsf{L}^{[c]}(\mathsf{L}^{[d]}(\mathsf{Bool}));$

$\{\mathbf{a}\otimes\mathbf{b}, \mathbf{a}\otimes\mathbf{c}, \mathbf{a}\otimes\mathbf{d}\} \vdash$ *separation pre-condition*

match x with
  Nil $\rightarrow y$
  | Cons(h, t)@$d \rightarrow$ let $z =$ Cons(h, y)@d
      in $\mathit{append}(t, z)$

$: \mathsf{L}^{[e\subseteq\{a,c\}]}(\mathsf{L}^{[f\subseteq\{b,d\}]}(\mathsf{Bool}));$ *containment guarantees*

$\{\mathbf{a}\};$ *destroyed portions*

$\otimes\mathbf{f}/\mathbf{e} \Longleftarrow \{\mathbf{b}\otimes\mathbf{d}, \otimes\mathbf{d}/\mathbf{c}\}$ *separation rely-guarantees*

Typing a recursive program

- annotating expressions - need annotated function symbols
- initial annotation - optimistic
- then annotate expression and repeat until fixpoint is found

$x : \mathsf{L}^{[a]}(\mathsf{L}^{[b]}(\mathsf{Bool})), y : \mathsf{L}^{[c]}(\mathsf{L}^{[d]}(\mathsf{Bool}));$

$\{\mathbf{a}\otimes\mathbf{b}, \mathbf{a}\otimes\mathbf{c}, \mathbf{a}\otimes\mathbf{d}\} \vdash$ *separation pre-condition*

match x with

  Nil $\longrightarrow y$

  | Cons(h, t)@$d \longrightarrow$ let $z =$ Cons(h, y)@d

  in $\mathit{append}(t, z)$

$: \mathsf{L}^{[e\subseteq\{a,c\}]}(\mathsf{L}^{[f\subseteq\{b,d\}]}(\mathsf{Bool}));$ *containment guarantees*

$\{\mathbf{a}\};$ *destroyed portions*

$\otimes\mathbf{f}/\mathbf{e} \Longleftarrow \{\mathbf{b}\otimes\mathbf{d}, \otimes\mathbf{b}/\mathbf{a}, \otimes\mathbf{d}/\mathbf{c}\}$ *separation rely-guarantees*

Typing a recursive program

- annotating expressions - need annotated function symbols
- initial annotation - optimistic
- then annotate expression and repeat until fixpoint is found

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$\{a \otimes b, a \otimes c, a \otimes d\} \vdash$ *separation pre-condition*

match x with

Nil $\rightarrow y$

$|$ Cons$(h, t)@d \rightarrow$ let $z =$ Cons$(h, y)@d$

in $append(t, z)$

$: \mathsf{L}^{[e \subseteq \{a,c\}]}(\mathsf{L}^{[f \subseteq \{b,d\}]}(\mathsf{Bool}));$ *containment guarantees*

$\{a\};$ *destroyed portions*

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  - Scope: allocation, higher order, arrays