Final report
on EPSRC Visiting Research Fellowship GR/M06468
supporting Yoram Hirshfeld

The object of Fellowship was to enable the nominated fellow, Yoram Hirshfeld, to visit Edinburgh to continue collaborative research on algorithmic aspects of concurrency theory. This work can be viewed as a natural extension to, and fusion of, classical formal language theory, logic and complexity theory. The specific aims of the visit, as presented in the proposal, were as follows:

(1) to bring to fruition a long-running project to develop the first decision procedure for bisimulation equivalence in "normed PA";

(2) to start a new attack on the model checking problem for the modal μ-calculus;

(3) to discuss possible directions for future work on *probabilistic* aspects of concurrent systems.

Aim (1) was accomplished, and the greater part of this report will be devoted to an explanation of the results obtained. Little or no progress was made towards aim (2), about which no more will be said here. After submitting the proposal, we became aware of existing (but recent) work connected with aim (3), and it took us longer to reach new territory than anticipated; some brief remarks on this topic will close the report.

The background to aim (1) is now sketched. Let $\mathcal{A}$ be a finite set of *atoms*, Act a finite set of *actions*, and Δ a collection of *productions* of the form $X \xrightarrow{a} Y$, where $X, Y \in \mathcal{A}$ and $a \in \text{Act}$. Regarding the atoms as states of a system, we can think of the production $X \xrightarrow{a} Y$ as specifying a possible evolution of the system from state X to Y via action a. What we have is nothing more than a finite state automaton, familiar from formal language theory.

We can generalise this situation by allowing both the states and the right hand sides of productions to be terms in the free algebra over $\mathcal{A}$ generated by an associative non-commutative operator "$\cdot$" that we think of as "sequential composition", and an associative commutative operator "$\mid$" representing "parallel composition". For example, if $W, X, Y, Z \in \mathcal{A}$, then $X \mid (Y \cdot Z)$ denotes a composite process in which X evolves in parallel with another process $Y \cdot Z$, which in turn evolves as Y followed by Z. The production $W \xrightarrow{a} X \mid (Y \cdot Z)$ expresses the fact that atomic process W may undergo action a to become the composite process $X \mid (Y \cdot Z)$. The derivation relation $\xrightarrow{a}$ extends to composite processes in a natural way: thus $W \cdot X \xrightarrow{a} (X \mid (Y \cdot Z)) \cdot X$ and $X \mid W \xrightarrow{a} X \mid X \mid (Y \cdot Z)$, but *not* $X \cdot W \xrightarrow{a} X \cdot (X \mid (Y \cdot Z))$ since $X \cdot W$ is sequential composition and X must be exhausted before W can start to evolve.

This set-up can be viewed as a fragment of the process algebra ACP, the *Algebra of Communicating Processes* of Bergstra and Klop; we refer to this fragment as *PA*. As a model for concurrent systems, PA still lacks the important element of synchronisation (the "C" in "ACP"), but at least it is a move in the right direction.

A prime concern in the theory of concurrent systems is to determine when two processes are "behaviourally equivalent", a notion made rigorous in Park's relation of bisimilarity. Two processes are *bisimilar* if, roughly, they may evolve together in such a way that whenever the first process performs a certain action, the second process is able to respond by performing the same action, and *vice versa*. An open problem of some years' standing is whether bisimilarity is decidable in the system PA. In the yet smaller fragment BPA, with just the sequential composition operator, bisimilarity was shown to be decidable by Christensen, Hüttel, and Stirling; the matching result for the fragment BPP, with just the parallel composition operator, is due to Christensen, Hirshfeld, and Moller.

The core of the problem with PA lies in the surprising interactions that can occur between sequential and parallel composition. In particular, there are situations in which the sequential composition of two processes $P_1 \cdot P_2$ may be equivalent to a parallel composition $Q_1 \mid Q_2$ of two other processes. A trivial example is given by $\mathcal{A} = \{X\}$, $\mathrm{Act} = \{a, b\}$ and productions $X \xrightarrow{a} X \mid X$ and $X \xrightarrow{b} \varepsilon$, for which system $X \cdot X$ is bisimilar to $X \mid X$. But this is just the simplest case, and the equivalence $P_1 \cdot P_2 \sim Q_1 \mid Q_2$ in fact has an infinite set of solutions of apparently unbounded complexity.

We (Hirshfeld and Jerrum) had been attempting to classify the situations in which a sequential composition of two processes could be bisimilar to a parallel composition, under the technical condition of processes being "normed". That classification has now been completed, with the support of the current VRF. Fortunately, the infinite collection of solutions mentioned earlier can be classified using a small finite number of patterns (applied recursively). As a consequence of this structure theory for PA (which may be of independent interest), we were able to exhibit the first decision procedure for bisimulation equivalence of normed PA [2].

Finally, some remarks on aim (3), which was to investigate logics for specifying and verifying properties of systems with probabilistic as well as nondeterministic choices. Our rough idea was to define a *probabilistic* modal μ-calculus, whose models are labelled transition systems with transition probabilities: essentially Markov decision processes. In fact, this programme turns out to be similar to ones already underway, e.g., at Birmingham (Kwiatkowska) and Stanford (de Alfaro [1]), based on probabilistic extensions of the logic CTL. We are now more in command of the topic, but we do not at this stage have results to report which are significantly distinct from those of other researchers.

References

[1] Luca de Alfaro, *Formal Verification of Probabilistic Systems*, Ph.D. Thesis, Department of Computer Science, Stanford University, December 1997.

[2] Yoram Hirshfeld and Mark Jerrum, Bisimulation equivalence is decidable for normed Process Algebra, *Internal Report ECS-LFCS-98-386*, Department of Computer Science, University of Edinburgh, May 1998. (The report may be downloaded from `http://www.dcs.ed.ac.uk/lfcsreps/`)