Report on the RAND-APX (Randomized and Approximate Computation) Workshop ICMS, Edinburgh, 17th–19th September 2000

RAND-APX ("Randomized and Approximate Computation") is a Thematic Network funded by the EU Esprit initiative, with sites in Bonn, Edinburgh, Leeds, Lund, Oxford, Paris, and the Weizmann Institute, Rehovot. The RAND-APX Thematic Network holds workshops once or twice a year, which cycle among the various sites. The most recent workshop took place in Edinburgh under the auspices of the International Centre for Mathematical Science (ICMS), and ran from Sunday 17th to Tuesday 19th September 2000. The workshop was open to participants from outside the Working Group, and the total number attending was about 25.

The RAND-APX Thematic Network has its origins RAND & RAND2 Working Groups, but widens and extends their scope to take account of recent developments. An innovation introduced by the RAND-APX Thematic Network is the assignment of themes to workshops, with "statistical physics" being chosen for the first meeting. The theme was reflected in the choice of invited speakers, two of whom were bona fide physicists. We were fortunate in obtaining four excellent invited speakers for the meeting: Marek Biskup, Roman Kotecký, Andrew Stuart and Prasad Tetali. In addition to the invited talks there was a high-quality programme of shorter contributed talks.

As usual, the ICMS's headquarters in India Street (the birthplace of James Clerk Maxwell) provided an pleasant venue for the meeting. The following list of abstracts serves as a partial record of the meeting.

Mark Jerrum

Invited talks

Statistical mechanics with complex weights: rigorous theory of partition function zeros

Marek Biskup (Microsoft Research, Redmond, WA, USA)

Abstract. In equilibrium statistical mechanics, partition function (or statistical sum) in volume V is the sum of the thermodynamic weight over all of the configurations in V. For many statistical-mechanical models on a lattice (e.g., the Ising, Blume-Capel or Potts models), the partition function is a polynomial in fugacity $z = e^{\beta h}$, where β is the inverse temperature and h is an external field. In my talk I will describe a recent new theory on the position of the zeros of the partition function for models with a convergent contour expansion. The aformentioned models are all in this class, provided the inverse temperature β is large enough.

For dimensions $d \geq 2$ and periodic volumes of size $V = L^d$, the technique I will outline allows localizing the zeros up to a precision of order e^{-cL} (with some $c > 0$), which is extremely small compared to the zeros' typical mutual distance ($\gtrsim 1/L^d$). Generally, the

zeros concentrate along the lines of complex phase transitions. If there is an exact symmetry with respect to circle inversion, the zeros may in places fall *exactly* on the unit circle. This is a *local version* of the classic Lee-Yang Circle Theorem[1]. I will conclude by mentioning some interesting open questions concerning lattice colorings whose resolution may require an approximate computation of the partition function zeros.

Joint work [2,3] with C. Borgs, J.T. Chayes, L.J. Kleinwaks, and R. Kotecký.

[1] T.D. Lee, C.N. Yang, *Phys. Rev.* **87**, (1952) 410.

[2] M. Biskup *et al.*, *Phys. Rev. Lett.* **84** (2000) 4794.

[3] M. Biskup *et al.*, in preparation.

Mathematics of phase transitions

Roman Kotecký (Charles University, Prague, Czech Republic)

Abstract. This is a review of some basic techniques from general mathematical theory of phase transitions of lattice models. It is shown how to rewrite the considered models in terms of contours and how to treat the resulting systems with help of contour expansions. In particular, Dobrushin's elegant proof of convergence of cluster expansions is reviewed and the main idea of Pirogov-Sinai theory is sketched.

[1] Roman Kotecký, *Phase transitions of lattice models* (Rennes lectures).
`http://www.cts.cuni.cz/~kotecky/publ/rennes.pdf`

Algorithms for collision detection amongst spheres

Andrew Stuart (Warwick, UK)

Abstract. The efficient calculation of collisions amongst a large number of linearly propagating spheres at constant speed is a well-studied problem in both the computer science and molecular dynamics literature. We review this work and describe some analysis which goes some way to unifying it. In particular we prove, under the Boltzmann-like assumptions on the statistics, that the expected complexity is near-optimal with optimal choice of the algorithmic parameters.

We then proceed to study the harder problem of calculating collisions between spheres moving along the trajectories of a given background field, possibly itself coupled back to the particle positions. Such problems arise in the simulation of sprays (modelled as particles) interacting with a fluid (the background field). Some algorithms are proposed and experiments shown to demonstrate their effectiveness. In particular, heuristics arising from the analysis of algorithms for a hard-sphere gas are shown to be useful.

Joint work with Hersir Sigurgeirsson and Justin Wan (Stanford).

Poincaré, Cheeger and log-Sobolev

Prasad Tetali (Georgia Tech, Atlanta, GA, USA)

Abstract. This is a survey talk on proving Cheeger-type inequalities relating Poincaré-type constants (such as spectral gap) to isoperimetric constants (such as expansion and conductance) in the (discrete) setting of graphs and finite Markov chains. The connection to and relevance of the finer log-Sobolev inequalities will also be described.

Contributed talks

Balanced allocations: The heavily loaded case or "balls-into-bins games"

Petra Berenbrink (Warwick, UK)

Abstract. We investigate load balancing processes based on the multiple-choice paradigm. In these randomized processes m balls are inserted into n bins. In the classical single-choice variant each ball is placed simply into a randomly selected bin. In a multiple-choice process each ball can be placed into one out of $d \geq 2$ randomly selected bins. It is well known that having more than one choice for each ball can improve the load balance significantly. In contrast to previous work on multiple-choice processes, we investigate the heavily loaded case, that is, we assume $m \gg n$ rather than $m \approx n$.

The best previously known results for the multiple-choice processes in the heavily loaded case were obtained by majorization from the single-choice process. This yields an upper bound of $m/n + O(\sqrt{m \ln n / n})$. We show, however, that the multiple-choice processes are fundamentally different from the single-choice variant in that they have "short memory". The great consequence of this property is that the deviation of the multiple-choice processes from the optimal allocation (i.e., at most $\lceil m/n \rceil$ balls in every bin) does not increase with the number of balls as in case of the single-choice process.

In particular, we investigate the allocation obtained by two different multiple-choice allocation schemes, the original greedy scheme and the recently presented always-go-left scheme. We show that these schemes result in a maximum load of only $m/n + O(\ln \ln n)$. We point out that our detailed bounds are tight up to additive constants.

Furthermore, we investigate the two multiple-choice algorithms in a comparative study. We present a majorization result showing that the always-go-left scheme obtains a better load balancing than the greedy scheme for any choice of n, m, and d.

Joint work with A. Czumaj, A. Steger and B. Vöcking.

Empirical risk approximation for data clustering

Joachim Buhmann (Bonn, Germany)

Abstract. Unsupervised learning algorithms are designed to extract structure from data samples. The most frequently extracted structures are compact groups of data. Reliable

and robust inference requires a guarantee that extracted structures are typical for the data source, i.e., similar structures have to be extracted from a second sample set of the same data source. Lack of robustness is known as overfitting from the statistics and the machine learning literature. In this paper the overfitting phenomenon is characterized for a class of histogram clustering models which play a prominent role in information retrieval, linguistic and computer vision applications. Learning algorithms with robustness to sample fluctuations are derived from large deviation theory and the maximum entropy principle for the learning process. The theory validates continuation methods like simulated or deterministic annealing as robust approximation schemes from a statistical learning theory point of view. It predicts the optimal approximation quality, parameterized by a minimal finite temperature, for given covering numbers of the hypothesis class for structures. Monte Carlo simulations are presented to support the underlying inference principle. Applications of the robust optimization principle is shown for the Euclidian Traveling Salesman Problem.

Hamilton cycles in random r-regular graphs for large r

Colin Cooper (Goldsmiths College, London)

Abstract. Let G_r denote a graph chosen uniformly at random from the set of r-regular graphs with vertex set $\{1, 2, \ldots, n\}$ where $10^6 \leq r \leq c_0 n$ for some small constant c_0. We prove that with probability tending to 1 as $n \to \infty$, G_r has the following properties: G_r is r-connected, and G_r is Hamiltonian.

Our method of proof uses the technique of edge swaps $ab, cd \to ac, bd$, both in the space of r-regular configurations, to estimate algorithmically the probability that a configuration multigraph is simple, and in the space of simple r-regular graphs to bound the co-degree of any two vertices.

The talk is an extract from the paper *Random regular graphs of non-constant degree* which also shows that for $r \leq n^{1-\eta}$, $r \to \infty$ (where $\eta > 0$ can be arbitrarily small) the independence number of G_r is approximately $(2n \log r)/r$ and the chromatic number of G_r is approximately $r/(2 \log r)$.

Joint work with A. Frieze and B. Reed.

Markov chains for random H-colourings

Martin Dyer (Leeds, UK)

Abstract. Let $H = (W, F)$ be a graph without multiple edges, but with the possibility of having loops. Let $G = (V, E)$ be a simple graph. A *homomorphism* c is a map $c : V \to W$ with the property that $(v, w) \in E$ implies $(c(v), c(w)) \in F$. We refer to $c(v)$ as the *colour* of v and c as an H-*colouring* of G. We consider the problem of choosing a random H-colouring of G by Markov Chain Monte Carlo, in particular by the *Glauber dynamics*, which modifies the colour of one vertex at a time. We give negative results for uniform sampling and a positive result for weighted sampling when H is a "looped tree". The model we consider includes

random proper colourings, random independent sets and the Widom-Rowlinson and Beach models o Statistical Physics.

Joint work with Colin Cooper and Alan Frieze.

On the relative complexity of approximate counting problems

Leslie Goldberg (Warwick, UK)

Abstract. Two natural classes of counting problems that are interreducible under approximation-preserving reductions are: (i) those that admit a particular kind of efficient approximation algorithm known as an "FPRAS," and (ii) those that are complete for #P with respect to approximation-preserving reducibility. We describe and investigate not only these two classes but also a third class, of intermediate complexity, that is not known to be identical to (i) or (ii). The third class can be characterised as the hardest problems in a logically defined subclass of #P.

Joint work with Martin Dyer, Catherine Greenhill and Mark Jerrum.

Improved algorithms for constructing sparse geometric spanners

Joachim Gudmundsson (Lund, Sweden)

Abstract. Let $G = (V, E)$ be a connected graph with positive weights and n vertices. A subgraph G' is a t-spanner if for all $u, v \in V$, the distance between u and v in the subgraph G' is at most t times the corresponding distance in G. We show an $O(n \log n)$-time algorithm which, given a set V of n points in d-dimensional space, and any constant $t > 1$, produces a t-spanner of the complete Euclidean graph of G. The produced spanner has $O(n)$ edges, constant degree and weight $O(wt(MST))$.

Joint work with Giri Narasimhan and Christos Levcopoulos.

Approximating the permanent (part I)

Mark Jerrum (Edinburgh, UK)

Abstract. This two-part presentation (with Vigoda) develops a fully-polynomial randomized approximation scheme for computing the permanent of an arbitrary matrix with non-negative entries. Part I sets the scene by reviewing an existing MCMC approach to approximating the permanent, proposed by Broder and made rigorous by Jerrum and Sinclair using the "canonical paths" argument. The limitations of the existing approach are described, and an obstacle to further progress identified.

Joint work with Alistair Sinclair.

Improved approximation of MAX-CUT on graphs of bounded degree

Marek Karpinski (Bonn, Germany)

Abstract. We design a new approximation algorithm for the MAX-CUT problem based on semidefinite relaxation of MAX-CUT, and random hyperplane rounding technique combined with a new local improvement placement method. Let $\alpha \simeq 0.87856$ denote the approximation factor of the best currently known approximation algorithm of Goemans and Williamson for the MAX-CUT problem on general graphs. We show that for graphs of degree at most Δ, our algorithm achieves an approximation factor of at least $\alpha + \varepsilon$, where $\varepsilon > 0$ is a constant depending only on Δ.

In particular, our algorithm achieves an approximation factor of at least 0.921 for graphs of maximal degree 3, and an approximation factor of at least 0.924 for 3-regular graphs.

Joint work with U. Feige and M. Langberg.

A PTAS for Max-bisection on planar graphs

Andrzej Lingas (Lund, Sweden)

Abstract. The Max-Bisection and Min-Bisection are the problems of finding partitions of the vertices of a given graph into two equal size subsets so as to maximize or minimize, respectively, the number of edges with exactly one endpoint in each subset.

In this paper we design the first polynomial time approximation scheme for the Max-Bisection problem on arbitrary planar graphs solving a long time standing open problem. The method of solution involves a design of exact polynomial time algorithms for computing optimal partitions of bounded treewidth graphs, in particular their Max- and Min-Bisections, which could be also of independent interest.

Joint work with Klaus Jansen and Marek Karpinski.

Discrete-time balanced allocations in fully-connected networks

Malwina Luczak (Oxford, UK)

Abstract. We consider a random allocations model related to the well-known "balls into bins" paradigm. Our setting is a complete graph on n vertices and a sequence of $\binom{n}{2}$ random calls arriving at discrete-time steps. We investigate the performance of two online routing algorithms.

Both strategies first try to route a new call directly. If there is no spare capacity on the direct path, then the Dynamic Alternative Routing algorithm sequentially checks up to d random two-link alternative routes, and assigns the call to the first path with free channels. The Balanced Dynamic Alternative Routing algorithm selects simultaneously, with replacement, d alternative two-link routes. The maximum loaded (in a suitable sense) edges on all the paths are compared and the call is routed along the path that minimises the maximum load.

We derive precise asymptotics of the blocking probability under the two regimes. This work extends and develops more fully the earlier paper by Luczak and Upfal (1999).

Joint with Colin McDiarmid.

Semantical counting circuits

Miklos Santha (Paris Sud, Orsay, France)

ABSTRACT. Counting functions can be defined syntactically or semantically depending on whether they count the number of witnesses in a non-deterministic or in a deterministic computation on the input. In the Turing machine based model, these two ways of defining counting were proven to be equivalent for many important complexity classes. In the circuit based model, it was done for #P and #L, but for low-level complexity classes such as $\#\mathrm{AC}^0$ and $\#\mathrm{NC}^1$ only the syntactical definitions were considered. We give appropriate semantical definitions for these two classes and prove them to be equivalent to the syntactical ones. This enables us to show that $\#\mathrm{AC}^0$ is included in the family of counting functions computed by polynomial size and constant width counting branching programs, therefore completing a recent result of Caussinus et al. We also consider semantically defined probabilistic complexity classes corresponding to AC^0 and NC^1 and prove that in the case of unbounded error, they are identical to their syntactical counterparts.

Joint work with Fabrice Noilhan.

Approximating the permanent (part II)

Eric Vigoda (Edinburgh, UK)

ABSTRACT. This two-part presentation (with Jerrum) develops a fully-polynomial randomized approximation scheme for computing the permanent of an arbitrary matrix with non-negative entries. Part II describes how to modify the existing Markov chain—by applying carefully chosen weights to configurations of unmatched vertices—in order to achieve rapid mixing for all problem instances. The weights may be approximated by an iterative procedure in which MCMC is used to adjust the weights at each step.

Joint work with Alistair Sinclair.