Esprit Working Group 21726, "RAND2"

Report on the Edinburgh Site, 1997–98

Overview of recent developments

Our main effort continues to be directed towards providing a rigorous foundation for approximation algorithms based on Markov chain simulation (sometimes termed "Markov chain Monte Carlo" or MCMC algorithms). The efficiency of such algorithms depends crucially on the number of steps which must be taken before the Markov chain it is close to equilibrium, the so-called "mixing time" of the Markov chain. Analysing the mixing time of Markov chains of practical interest remains a challenging activity, though the so-called "canonical paths" technique has scored some notable successes. Perhaps because of these successes, we have tended to concentrate on the canonical path method to the exclusion of others.

In the last RAND report I noted a tentative return to more classical "coupling" methods for analysing the mixing rate. Interest in coupling has continued to grow, stimulated by two recent developments: Propp and Wilson's method[1] for exact sampling using "coupling from the past" (CFP) and Bubley and Dyer's powerful "path coupling" technique[2] for constructing complex couplings. Gore and Jerrum have performed some experiments with an extension of CFP to non-monotone systems, due to Wilfrid Kendall. Some very preliminary results were reported at DIMACS, and will appear in a survey of recent developments in coupling techniques, currently in preparation. The limits of the path coupling method are currently being explored, but already the Leeds site has achieved some notable results.

Recent progress on *negative results* suggests an interesting direction for future work. (A negative result in this context amounts to establishing a lower bound on mixing time for some Markov chain.) Gore and Jerrum observed that a "first-order phase transition" in a physical system may be associated with a long mixing time [2]. In a related development, Dyer, Frieze and Jerrum combined the notion of first-order phase transition with another—probabilistically checkable proof or PCP—to obtain possibly the first non-trivial non-approximability result for a counting problem (paper in preparation).

Personnel

Mark Jerrum, Vivek Gore (until November 1997), Roni Khardon (from October 1997), Jitka Stříbrná (postgraduate student), Leslie Goldberg (Department of Computer Science, University of Warwick).

[1] James G. Propp and David B. Wilson, Exact sampling with coupled Markov chains and applications to statistical mechanics, *Random Structures and Algorithms* **9** (1996), 223–252.

[2] Russ Bubley and Martin Dyer, Path coupling: a technique for proving rapid mixing in Markov chains, *Proceedings of the 38th IEEE Symposium on Foundations of Computer Science*, 1997, 223–231.

Publications (since October 1996)

1. Leslie Goldberg and Mark Jerrum, Randomly sampling molecules, *Proceedings of the 8th ACM-SIAM Symposium on Discrete Algorithms*, 1997, 183–192.

2. Leslie Ann Goldberg and Philip D. MacKenzie, Contention Resolution with Guaranteed Constant Expected Delay, *Proceedings of the 38th IEEE Symposium on Foundations of Computer Science*, 1997, 213-222.

3. Leslie Ann Goldberg, Mike Paterson, Aravind Srinivasan and Elizabeth Sweedyk, Better approximation guarantees for job-shop scheduling, *Proceedings of 8th ACM/SIAM Symposium on Discrete Algorithms*, 1997, 599–608.

4. Vivek Gore and Mark Jerrum, *The Swendsen-Wang process does not always mix rapidly*, Internal Report ECS-LFCS-96-349, Department of Computer Science, University of Edinburgh, November 1996. Journal version in preparation.

5. Vivek Gore, Mark Jerrum, Sampath Kannan, Z. Sweedyk and Steve Mahaney, A quasi-polynomial-time algorithm for sampling words from a context-free language, *Information and Computation* **134** (1997), 59–74.

6. Vivek Gore and Mark Jerrum, The Swendsen-Wang process does not always mix rapidly, *Proceedings of the 29th ACM Symposium on Theory of Computation*, ACM Press, 1997, 674–681.

7. Russ Bubley, Martin Dyer and Mark Jerrum, An elementary analysis of a procedure for sampling points in a convex body. To appear in *Random Structures and Algorithms.*

Research Visits (since October 1996)

- 21st March 1997 to 7th June 1997 (except for the one week noted in the following item): Visit to DIMACS, Rutgers University, New Jersey, for the "Focus on Discrete Probability" (Mark Jerrum).

- 21st–26th April: Visit to Carnegie Mellon University, for collaborative research with Alan Frieze and Ravi Kannan (Jerrum).

Conferences/workshops attended (since October 1996)

- ACM/SIAM Symposium on Discrete Algorithms, New Orleans, January 1997. Contributed paper: "Randomly sampling molecules" (Mark Jerrum, with coauthor Leslie Goldberg).

- DIMACS workshop *Statistical Physics Methods in Discrete Probability, Combinatorics and Computer Science*, March 1997. "The Swendsen-Wang process does not always mix rapidly" (Jerrum, with coauthor Vivek Gore).

- DIMACS workshop *Probabilistic Analysis of Algorithms*, May 1997. Talk: "Data-sensitive analysis and a posteriori performance guarantees" (Jerrum).

- DIMACS workshop *Microsurveys in Discrete Probability*, June 1997 (Jerrum).

- *British Combinatorial Colloquium*, Queen Mary College, London, July 1997 (Jerrum).

- *Random Structures and Algorithms*, Poznań, Poland, August 1997. Invited talk: "Efficient sampling" (Jerrum); Contributed talk: "Random waiting schemes: protocols for resolving contention" (Goldberg).

- *Combinatorial Approximation Algorithms Workshop*, Schloß Dagstuhl, August 1997. Talk: "Approximate counting: a short introduction" (Jerrum).

- *RAND2 Meeting*, Leeds, September 1997. Talk: "Experiments in exact sampling" (Jerrum); participant (Goldberg).

- *Random Graphs and Combinatorial Structures* meeting, Oberwolfach, October 1997. Talk: "The Swendsen-Wang process does not always mix rapidly" (Jerrum, with coauthor Gore)

- *IEEE symposium on Foundations of Computer Science*, Miami, USA. Talk: "Contention Resolution with Guaranteed Constant Expected Delay" (Goldberg, with coauthor Philip MacKenzie).