Esprit Working Group 21726, “RAND2”

Report on the Edinburgh Site, 1997–98

Overview of recent developments

Last year’s report mentioned *non-approximability* results for counting problems as an interesting direction for future work. A strong form of non-approximability result is one that rules out the existence of *any* efficient approximation algorithm—in the sense of “Fully Polynomial Randomised Approximation Scheme”, or FPRAS—under some reasonable complexity-theoretic assumption. Dyer, Frieze and Jerrum [2] combined the notion of first-order phase transition with another—probabilistically checkable proof or PCP—to obtain a non-approximability result for counting independent sets in a degree-bounded graph. This is an encouraging development in an area lacking non-trivial techniques, and points to a line of enquiry we intend to pursue in the coming year.

Given the paucity of strong non-approximability results, it is of interest to consider weaker forms of evidence. One way to proceed is to show that a certain natural approximation technique is doomed to failure. In the area of counting problems, this “natural technique” is likely to be the “Markov chain Markov chain” (MCMC) method, since this has proved the most fruitful approach. Sometimes the existence of a “phase transition” for the structures being enumerated can be used to rule out a wide range of potential MCMC attacks [2, 6].

A very different line of enquiry that has been actively pursued in the last year (by Goldberg, Jerrum, and others outside RAND2) is the theoretical study of contention resolution protocols for Ethernet channels. We have focused on classes of simple protocols, in which the probability of attempted transmission of a message is dependent only on its age (“age-based protocols”) or only on its count of failed attempts (“backoff protocols”). A number of apparently simple questions are still open—e.g., does there exist a “stable” backoff protocol in the “infinitely-many users” model, where the number of arrivals in each time step is distributed as a Poisson random variable—and we are actively working on these.

Personnel

Mark Jerrum, Vivek Gore (until November 1997), Roni Khardon, Jitka Stříbrná (postgraduate student, until September 1998), Leslie Goldberg (Department of Computer Science, University of Warwick).

Publications

1. Russ Bubley, Martin Dyer and Mark Jerrum, An elementary analysis of a procedure for sampling points in a convex body, *Random Structures and Algorithms* **12** (1998), 213–235.

2. Martin Dyer, Alan Frieze and Mark Jerrum, On counting independent sets in sparse graphs, *Internal Report ECS-LFCS-98-391*, Department of Computer Science, University of Edinburgh, July 1998. Submitted to *SIAM Journal on Computing.*

3. Martin Dyer, Alan Frieze, and Mark Jerrum, Approximately counting Hamilton paths and cycles in dense graphs, *SIAM Journal on Computing* **27** (1998), 1262–1272.

4. Alan Frieze and Mark Jerrum, Improved approximation algorithms for MAX k-CUT and MAX BISECTION, *Algorithmica* **18** (1997), 67–81.

5. Leslie Ann Goldberg, Paul W. Goldberg, Cynthia A. Phillips and Gregory B. Sorkin, Constructing computer virus phylogenies, *Journal of Algorithms* **26**, (1998) 188–208.

6. Leslie Ann Goldberg and Mark Jerrum, The "Burnside process" converges slowly, *Internal Report ECS-LFCS-98-387*, Department of Computer Science, University of Edinburgh, June 1998. (Also *Computer Science Research Report CS-RR-344*, University of Warwick, June 1998. Preliminary version in: Randomization and Approximation Techniques in Computer Science (M. Luby, J. Rolim and M. J. Serna, eds.), *Springer-Verlag LNCS* **1518**, 1998, 331–345. Journal version submitted to *Combinatorics, Probability and Computing.*)

7. Leslie Ann Goldberg, Mark Jerrum, Tom Leighton, and Satish Rao, Doubly logarithmic communication algorithms for optical communication parallel computers, *SIAM Journal on Computing* **26** (1997), 1100-1119.

8. Leslie Ann Goldberg, Mark Jerrum, and Philip D. MacKenzie, An $\Omega(\sqrt{\log\log n})$ lower bound for routing in optical networks, *SIAM Journal on Computing* **27** (1998), 1083–1098.

9. Yoram Hirshfeld and Mark Jerrum, Bisimulation equivalence is decidable for normed Process Algebra, *Internal Report ECS-LFCS-98-386*, Department of Computer Science, University of Edinburgh, May 1998.

10. Mark Jerrum, Mathematical foundations of the Markov chain Monte Carlo method. In *Probabilistic Methods for Algorithmic Discrete Mathematics* (M. Habib, C. McDiarmid, J. Ramirez-Alfonsin & B. Reed, eds), Algorithms and Combinatorics **16**, Springer-Verlag, 1998, 116–165.

11. Mark Jerrum and Gregory B. Sorkin, The Metropolis algorithm for graph bisection, *Discrete Applied Mathematics* **82** (1998), 155–175.

Research Visits

- One week visit by Mark Jerrum to the Department of Computer Science, University of Warwick, for research collaboration, April 1998. Main topic: mixing rate of the "Burnside process".

- Research visit by Leslie Goldberg to DIMACS to visit Martin Farach and Sampath Kannan, May 1998 (2 weeks). Research discussed: contention resolution, stochastic methods for constructing evolutionary trees, the shortest common superstring problem, stochastic problems concerning communication over noisy channels.

- One week research visit to University of Warwick by Mark Jerrum and Sampath Kannan, August 1998. Main topic: contention resolution in Ethernet channels.

Conferences/workshops attended

- RAND2 Meeting, Edinburgh, March 1998. Leslie Goldberg (speaker: "Contention resolution in multiple-access channels"); Mark Jerrum (local organiser).

- 14th British Colloquium for Theoretical Computer Science (BCTCS14), St Andrews, March/April 1998. Leslie Goldberg (invited speaker: "Contention resolution in multiple-access channels").

- Reading One-Day Combinatorics Colloquium, Reading, May 1998. Leslie Goldberg (invited speaker: "Uniform sampling modulo symmetries").

- First meeting of the Erdős Institute, Budapest, July 1998. Mark Jerrum (participant).

- WRASS (Warwick Randomised Algorithms and Stochastic Simulation Tutorial and Workshop), Warwick University, July 1998. Mark Jerrum (invited speaker, "Torpid Mixing and Non-Approximability"); Leslie Goldberg (organiser, tutorial: "Markov Chain Stability and Contention Resolution").

- Summer school on Probabilistic Methods for Algorithmic Discrete Mathematics, Montpellier, August 1998. Mark Jerrum (series of invited lectures on "Rapidly mixing Markov Chains").

- 2nd International Workshop "Random '98", Barcelona, October 1998. Leslie Goldberg (speaker: "The 'Burnside process' converges slowly").