Ordinary combinators

$$\begin{array}{llllllll}
C & f & a & b & \mapsto & f & b & a \\
B & f & a & b & \mapsto & f & (a\ b) & \\
W & f & a & & \mapsto & f & a & a \\
I & f & & & \mapsto & f & & \\
K & f & a & & \mapsto & f & &
\end{array}$$

C	swaps/interchanges arguments
B	composition (usual way round)
W	duplication of argument (idempotence)
K	discards argument

Arithmetical combinators

$$
\begin{array}{llllllll}
A & a & b & c & d & \mapsto & b & c & (a\ c\ d) \\
M & a & b & c & & \mapsto & b & (a\ c) & \\
E & a & b & & & \mapsto & b & a & \\
N & a & b & & & \mapsto & b & &
\end{array}
$$

$E = \wedge$	backwards application
$M = \times$	(forwards) composition (like ';')
$A = +$	pointwise version of $\times$
$N = 0$	discard argument

Notation

Operators

$$\begin{aligned} A &\rightarrow (+) \\ M &\rightarrow (\times) \\ E &\rightarrow (\wedge) \\ N &\rightarrow \ 0 \end{aligned}$$

Section notation

$$\begin{array}{ccccccccc} (\wedge)\ a\ b & = & (a\wedge)\ b & = & a \wedge b & = & b \wedge a \wedge (\wedge) & = & b \wedge (a\wedge) \\ (\times)\ a\ b & = & (a\times)\ b & = & a \times b & = & b \wedge a \wedge (\times) & = & b \wedge (a\times) \\ (+)\ a\ b & = & (a+)\ b & = & a + b & = & b \wedge a \wedge (+) & = & b \wedge (a+) \end{array}$$

Opportunity for $\Sigma_i \alpha_i$, $\Pi_i \alpha_i$.

Reduction of BWICK to AMEN

C	a	b	c	$=$	a	c	b	
				$=$	E	b	$(a\ c)$	$(c \wedge a) \wedge (b \wedge)$
C	a	b		$=$	M	a	$(E\ b)$	$a \times (b \wedge)$
C	a			$=$	M	E	$(M\ a)$	$(\wedge) \times (a \times)$
C				$=$	M	M	$(M\ E)$	$(\times) \times ((\wedge) \times)$
B				$=$	C	M		$(\wedge) \times ((\times) \times)$
K				$=$	C	N		$(\wedge) \times (0 \times)$
I				$=$	N	N		$0 \wedge 0 = (\wedge) \times ((\wedge) \times)$
W				$=$	C	V		$(\wedge) \times (((\wedge) + (\wedge)) \times)$
V	a	f		$=$	f	a	a	
				$=$	E	a	$(E\ a\ f)$	$f \wedge ((a \wedge) \times (a \wedge))$
V				$=$	A	E	E	$(\wedge) + (\wedge)$

Point out that KIW is a series. Note $0 \wedge 0 = ((\wedge)\times) \wedge ((\wedge)\times)$.

Exponents, logarithms, algebra

$\gamma \wedge (\alpha + \beta) = (\gamma \wedge \alpha) \times (\gamma \wedge \beta)$	$\gamma \wedge 0 = 1$
$\gamma \wedge (\alpha \times \beta) = (\gamma \wedge \alpha) \wedge \beta$	$\gamma \wedge 1 = \gamma$
$\log_\gamma (A \times B) = \log_\gamma A + \log_\gamma B$	$\log_\gamma 1 = 0$
$\log_\gamma (A \wedge \beta) = (\log_\gamma A) \times \beta$	$\log_\gamma \gamma = 1$
$\alpha \times 1 = \alpha = 1 \times \alpha$	
$\alpha \times (\beta \times \gamma) = (\alpha \times \beta) \times \gamma$	
$\alpha + 0 = \alpha = 0 + \alpha$	
$\alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma$	
$\alpha \times (\beta + \gamma) = \alpha \times \beta + \alpha \times \gamma$	
$\alpha \times 0 = 0$	

Exponentiality:

$$(\forall \gamma.\ \gamma \wedge \alpha = \gamma \wedge \beta) \Rightarrow \alpha = \beta$$

$$\log_\gamma(\alpha_1 \wedge \cdots \wedge \alpha_k \wedge A \wedge \beta) = (\log_\gamma A) \times \beta \times (\alpha_k \wedge) \times \cdots \times (\alpha_1 \wedge)$$