UNIVERSITY OF EDINBURGH

FACULTY OF SCIENCE AND ENGINEERING

MASTER OF SCIENCE IN COMPUTER SCIENCE

FORMAL PROGRAMMING LANGUAGE SEMANTICS

Tuesday 17 April 2001

09:30 to 11:00

Examiners:

I. Watson (External)

R.N. Ibbett (Chairman)

INSTRUCTIONS TO CANDIDATES

Answer any TWO questions.

All questions carry equal weight.

1. Consider the following syntax for commands in a simple while language.

$$c \;::=\; X := a \;\mid\; \textbf{if } b \textbf{ then } c_1 \textbf{ else } c_2 \;\mid\; c_1 \,;\, c_2 \;\mid\; \textbf{while } b \textbf{ do } c$$

Here X ranges over a set of "locations", a ranges over a predefined set of arithmetic expressions, and b ranges over a predefined set of boolean expressions. We use $\sigma, \sigma', \ldots$ to range over the set of *states*, i.e. over the set of functions mapping locations to integers.

(a) Assume that evaluation relations of the form

$$\begin{array}{rcl} \langle a, \sigma \rangle & \longrightarrow & n \\ \langle b, \sigma \rangle & \longrightarrow & t \end{array}$$

have been defined for arithmetic and boolean expressions, where n and t range over integers and boolean truth values respectively. Give a set of proof rules for deriving an evaluation relation for commands of the form

$$\langle c, \sigma \rangle \;\longrightarrow\; \sigma'$$

[10 marks]

(b) Let P be the following program in the language:

$$\begin{array}{l} Y := 0\,; \\ Z := 0\,; \\ \textbf{while } Y < X \textbf{ do} \\ \qquad\qquad Y := Y + 1\,; \\ \qquad\qquad Z := Y - Z \end{array}$$

Give a detailed verification of the partial correctness assertion:

$$\{X = x,\ x \geq 0\} \quad P \quad \{Z = \lceil x/2 \rceil\}$$

(where $\lceil x/2 \rceil$ means the smallest integer n such that $n \geq x/2$), using the standard proof rules of Hoare logic. [15 marks]

2. Throughout this question, suppose that $(D, \sqsubseteq_D)$ and $(E, \sqsubseteq_E)$ are *complete pointed partial orders (cppos)*.

   (a) Define what it means for a function $f : D \to E$ to be continuous. Give full definitions of any auxiliary concepts you use. [7 marks]

   (b) Given a continuous function $f : D \to D$, what is meant by a *least fixed point* of f? [3 marks]

   (c) Give a detailed proof that every continuous function $f : D \to D$ has a unique least fixed point. [15 marks]

3. This question concerns exact real number computation.

   (a) Briefly explain how a datatype **stream** of infinite streams of integers can be used to represent real numbers in the interval $\mathbb{I} = \{x \in \mathbb{R} \mid -1 \leq x \leq 1\}$ in a way that supports the implemention of exact real arithmetic. [5 marks]

For a real number x, write $I(x)$ for the value

$$I(x) \quad = \quad \begin{cases} 1 & \text{if } 1 \leq x \\ x & \text{if } -1 \leq x \leq 1 \\ -1 & \text{if } x \leq -1 \end{cases}$$

Using an informal functional "pseudocode" notation, define functions on **stream** that represent the following functions, of one or two arguments, on $\mathbb{I}$.

(b) $x \;\mapsto\; x/2$ [2 marks]

(c) $x \;\mapsto\; -x$ [2 marks]

(d) $x \;\mapsto\; I(1+x)$ [3 marks]

(e) $x \;\mapsto\; I(2x)$ [4 marks]

(f) $x, y \;\mapsto\; (x+y)/2$ [8 marks]

(g) $x, y \;\mapsto\; I(x+y)$ [1 mark]

(Here, e.g., $x, y \;\mapsto\; I(x+y)$ means the function mapping its two arguments x and y to the value $I(x+y)$.)